Intro Video: Section 5.4 Indefinite Integrals and the Net Change Theorem

Math F251X: Calculus 1

Indefinite integrals

Recall FTC 2: $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b)-F(a)$

where $F(x)$ is any antiderivative of $f(x)$.

that is, $\int_a^b F'(x)\,dx = F(b)-F(a)$

Notation for a <u>generic</u> antiderivative of a function:

$\int f(x)\,dx = F(x) \Rightarrow F(x)$ is a generic antiderivative of $f(x)$

$\longrightarrow$ $\boxed{\int_a^b f(x)\,dx}$ is a number

Definite integral

$\longrightarrow$ $\boxed{\int f(x)\,dx}$ is a (family of) function(s)

"Indefinite integral"

Example

① $\int \sin(x) - x^{2.7}\,dx$

$$= -\cos(x) - \frac{x^{2.7+1}}{2.7+1} + C$$

$$= -\cos(x) - \frac{x^{3.7}}{3.7} + C$$

② $\int a + bx^2\,dx$

$$= ax + \frac{bx^3}{3} + C$$

Net Change Theorem

→ What does $\int_a^b F'(t)\,dt$ <u>mean</u>?

$$\int_a^b F'(t)\,dt = F(b) - F(a)$$

→ Net change of $F(t)$ on the interval $[a,b]$

<u>Example</u>: If $v(t) = \frac{ds}{dt}$ is the velocity of a particle,

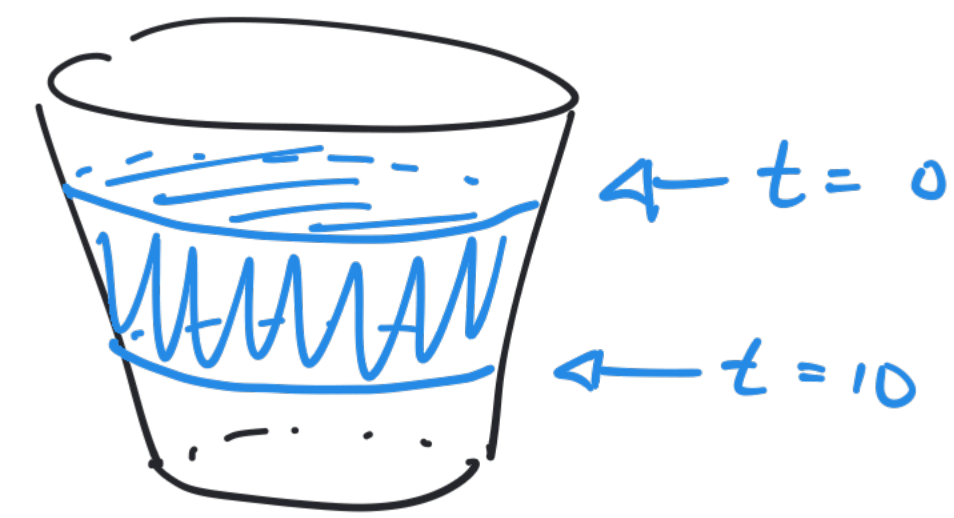
$\int_0^2 v(t)\,dt$ = change in position over the interval $[0,2]$.

Example: Water flows from a tank at a rate of

$$r(t) = 200 - 4t$$

liters/minute, for $0 \le t \le 50$. Find the amount of water that flows from the tank over the first 10 minutes.

Let $A(t)$ = amount of water in the tank at time t



The amount of water that flowed from the tank in the first 10 minutes

$$= \int_0^{10} r(t)\,dt = \int_0^{10} 200 - 4t\,dt = \left(200t - \frac{4t^2}{2}\right)\Big|_0^{10}$$

$$= \left[200(10) - 2(10)^2\right] - \left[200(0) - 2(0)^2\right]$$

$= 2000 - 2(100) = 800$ liters ← but we do not know $A(10)$